

To the blessed memory
of the remarkable mathematician Promarz Tamrazov

Functions with Generalized Derivatives on Anisotropic Spaces ¹

Vladimir M. Miklyukov

Below we introduce and study analogous of Sobolev function classes W_p^1 [1] on domains in anisotropic spaces. The investigation is based on the technology of modulus of curvilinear arcs families, developed in the theory of quasiconformal mappings (see, for example, [2]), and it is the most principal difference from articles based on integral representations techniques [3], capacity techniques [4] or closed to it [5].

- 1 Anisotropic Space $\mathcal{X} = \mathcal{X}_{r,\mu}$
- 2 Modulus of Curvilinear Families
- 3 Functions of the Class $\text{Sob}^{1,p}$
- 4 Embedding Theorems
 - 4.1 Conditions of Function Boundedness
 - 4.2 Conditions of Membership to the Class $L^q(U)$
 - 4.3 Hölder Continuity
- 5 Local Approximation
 - 5.1 Ends of Domains
 - 5.2 Setting of the Problem
 - 5.3 Main Theorem
- 6 Differentiability at Point
 - 6.1 Foliation
 - 6.2 Differentiability

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Vladimir M. Miklyukov,
Independent Scientific Laboratory "UCHIMSYA, LLC"
Yonkers, NY, USA
miklyuk@mail.ru, miklyuk@hotmail.com